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Energy losses of non-Newtonian fluids in sudden pipe contractions

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ABSTRACT

There are very few experimental results available for flow of viscous fluids in sudden contractions. Furthermore, there are discrepancies between the few sets of experimental data published so far. The objective of this work is to provide a set of reliable experimental results that confirms and extends existing datasets for experimental and numerical validation and design purposes.

The Flow Process Research Centre of the Cape Peninsula University of Technology has built and commissioned a test rig to study flows of both Newtonian and non-Newtonian fluids through sudden contractions of three diameter ratios β = 0.22, 0.5 and 0.85. This work extends the range of contraction ratios tested in literature from β = 0.66 to 0.85. The test fluids were water, glycerol solutions, lubrication oil, kaolin suspensions and carboxymethylcellulose (CMC) solutions at various concentrations. The Reynolds numbers ranged from approximately 0.01 to 100 000.

The experimental data agrees with a mechanistic analytical model, based on another set of experimental data in the literature, thereby demonstrating the credibility of our experimental results. A database of the results is available for these contraction ratios for experimental and numerical validation.

1. Introduction

Laminar flow through pipe fittings is still a topic that needs more investigation [1]. Most experimental studies on this topic include fittings such as contractions, expansions, elbows, valves and orifices [2–4]. McNeil and Morris [5] investigated flow through sudden contractions and expansions because understanding flow behaviour through these geometrically simple fittings could enhance the understanding of flow through more complex fittings such as valves, which are a combination of contracting and expanding flows. Turian et al. [3] showed that the loss coefficient is the same for Newtonian and non-Newtonian fluids in turbulent flow.

Judging from the volume of work done on flow through sudden contractions over the last 50 years [6–11], this topic definitely has a place in the fundamental understanding of fluid flow and fluid mechanics. Much of the early work was done to understand entry flows, focussing on downstream flow phenomena in turbulent flow [12–14].

In 1987, Boger [9] stated that laminar flow of Newtonian fluids through sudden contractions "is a solved problem". Sisavath et al. [11] indicated that several studies were done on laminar flow of non-Newtonian fluids through sudden contractions, but that less was done on Newtonian fluids. These authors stated that the determination of the additional pressure drop for laminar flow of Newtonian fluids through sudden contractions "is far from being resolved". The reason for this apparent contradiction was the fact that most experimental studies did not agree with one another or with analytical and numerical studies.

This work intended to redeem this view of experimental work by measuring a wide range of fluids in three contraction ratios that extends the available range to date and to find agreement amongst experimental data and semi-empirical models available in literature. The objective of this paper is to address these fundamental issues, and produce a set of reliable data, which can be used with confidence for validation and design purposes. This data is now available for experimental and numerical validation.

The energy losses across a piping system can be accounted for using the Bernoulli equation [15]. Hence, the loss coefficient of the fitting is given by

$$k_{\text{fitt}} = h_{\text{fitt}} \frac{2g}{V^2} \tag{1}$$

This can also be expressed in terms of pressure drop;

$$k_{\text{fitt}} = \frac{\Delta p_{\text{fitt}}}{(1/2)\rho V^2} \tag{2}$$

where k_{fitt} is the non-dimensionalised difference in overall pressure between the ends of two long straight pipes when there is no fitting and when the real fitting is installed [16], Δp_{fitt} is the pressure loss across the fitting and V is the mean flow velocity in the pipe.

Eq. (2) is based on the static pressure only. In the case of contractions where the upstream and downstream velocities are different,

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Nomenclature

A	cross-sectional area (m^2)
C_{con}	laminar flow loss coefficient constant (Couette)
D	internal pipe diameter (m)
E	sum of mean error squared
f	Fanning friction factor
g	gravitational acceleration (m/s^2)
h	head (m)
k	hydraulic roughness (m)
k_{con}	loss coefficient in contraction
k_{fitt}	loss coefficient in any fitting
K	fluid consistency index (Pa s^n)
K'	apparent fluid consistency index (Pa $s^{n'}$)
L	pipe length (m)
n	flow behaviour index
n'	apparent flow behaviour index
N	total number of
p	pressure (Pa)
Q	volumetric flow rate (m^3/s)
Re	Reynolds number
Re_3	Slatter Reynolds number (yield pseudoplastic)
Re_{crit}	critical Reynolds number at onset of transition
Re_{MR}	Metzner & Reed Reynolds number (Power law)
Re_N	Newtonian Reynolds number
V	average velocity (m/s)
z	elevation from datum (m)

Greek letters

α	kinetic energy correction factor
β	contraction ratio, D_d/D_u
$\dot{\gamma}$	shear rate (s^{-1})
Δ	increment
μ	dynamic viscosity (Pa s)
ρ	fluid or slurry density (kg m^{-3})
τ	shear stress (Pa)
τ_y	yield stress (Pa)

Subscripts

0	at the wall of the pipe
1	upstream of contraction
2	downstream of contraction
3	reference to Slatter Reynolds number
ann	annulus
calc	calculated
con	contraction
crit	critical
d	downstream
fitt	fitting
l	loss
m	mixture (slurry)
obs	observed (experimental)
plug	unsheared core in Re_3
r	roughness
s	solids
u	upstream

the total pressure should be used to account for changes in kinetic energy. Therefore, for a sudden contraction, Eq. (2) becomes [17], where α is the kinetic energy correction factor and Δp_{con} is the excess pressure drop:

$$k_{con} = \frac{[(\Delta p_{con}/\rho g) + (\alpha_1 V_1^2 - \alpha_2 V_2^2)/2g]}{{V_2}^2/2g} \tag{3}$$

The data are usually presented as plots of loss coefficient versus Reynolds number. The Reynolds number that best describes the viscous properties of the fluid should be used. For Newtonian fluids, the Newtonian Reynolds number will be used, for pseudoplastic fluids the Metzner & Reed Reynolds number, Re_{MR} [18] will be used and for yield pseudoplastic fluids the Slatter Reynolds number, Re_3 [19] will be used. The latter two Reynolds numbers are defined in Eqs. (4) and (5):

$$Re_{MR} = \frac{8\rho V^2}{K'(8V/D)^{n'}} \tag{4}$$

$$Re_3 = \frac{8\rho V_{ann}^2}{\tau_y + K(8V_{ann}/D_{shear})^n} \tag{5}$$

The relationships between K' and K and n' and n for a power-law fluid are given as [18]:

$$n' = n \text{ and } K' = K\left(\frac{1+3n}{4n}\right)^n$$

For a Newtonian fluid, $n' = 1$ and $K' = \mu$.

2. Experimental procedure

2.1. Apparatus

The experimental apparatus consisted of a 50 mm progressive cavity positive displacement pump, two flow meters (a magnetic flow meter and a corriolis mass flow meter), a 42.26 mm internal diameter upstream straight pipe section and a specially machined contraction union allowing for the fitting of straight pipe sections of various diameters downstream. The 200-l storage tank was fitted with a mixer driven by a motor of 1.5 kW to ensure that solids were kept in suspension during the experiments with slurries. The pump was fitted with a variable speed drive to enable tests at different flow rates. The fluid passed through a heat exchanger followed by a surge damper 50 mm in diameter and 0.4 m high. The fluid then passed through the electromagnetic flow meter and the mass flow meter, connected in series. The low-flow cut-off is 0–9.9% of the maximum flow rate. The upstream and downstream straight pipe sections were each 5 m long to ensure fully developed flow at the contraction inlet and redevelopment downstream of the contraction. The pipes were fitted with pressure tappings to measure the pressure grade line across the contractions.

All pressure tappings were connected to 3-mm nylon tube pressure lines filled with water. Nine point pressure transducers (PPT) and a differential pressure transducer (DP cell) were used to measure the static pressure along the length of the pipe and the differential pressure between two points, respectively. Both had a maximum range of 130 kPa with an accuracy of 0.25%. The ranges and span are adjustable electronically with a hand-held communicator to a turndown ratio of 10. A mercury–water manometer was used for calibration of the pressure transducers in the higher pressure range (≥20 kPa) and a water–air manometer was used to calibrate in the lower pressure range (≤20 kPa).

The upstream pipe diameter was kept constant while the downstream pipe was replaced by one of different diameters. A specially machined union was used to construct the sudden contraction. The pipes were mounted flush leaving no gaps that would influence the flow patterns. The upstream and downstream pipe diameters and the resulting contraction ratio in both diameter and area ratio of downstream to upstream pipes are shown in Table 1.

Fluids were selected that exhibit Newtonian, pseudoplastic and yield pseudoplastic behaviour to demonstrate that dynamic similarity can be obtained at the same Reynolds number, provided that the Reynolds number correctly accounts for the viscous properties

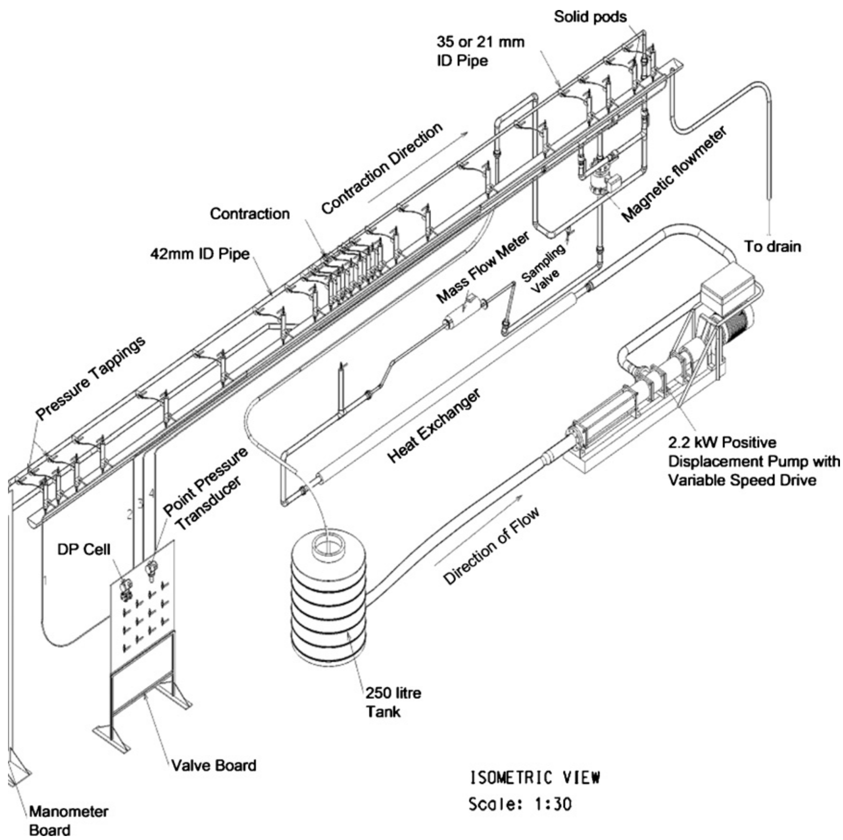

Fig. 1. Schematic diagram of experimental test rig.

of the fluid [20]. The fluids selected for the tests in this investigation were water, glycerol solutions, oil (Newtonian behaviour), carboxymethylcellulose (CMC) (pseudoplastic behaviour) and kaolin slurries (yield pseudoplastic behaviour) as previously shown in literature [2,3,20]. The properties of the fluids were determined using tube or rotary viscometry and the results are presented in the database.

2.2. Calculated and combined errors

The errors associated with the Reynolds number and loss coefficient were determined using the Brinkworth approach [21]. The Reynolds numbers used can be rewritten generically as

$$Re_{\text{gen}} = \frac{8\rho V^2}{\tau_0} \tag{6}$$

Prior to differentiation it is prudent to reduce (6) to its independent primary variables and can be written as

$$Re_{\text{gen}} = \frac{8\rho(Q/(\pi D^2/4))^2}{D\Delta P/4L} \tag{7}$$

resulting in

$$Re_{\text{gen}} = \frac{512\rho Q^2 L}{\pi^2 D^5 \Delta P} \tag{8}$$

Applying Brinkworth approach to Eq. (8) gives Eq. (9) which after differentiation and simplification yield Eq. (10):

$$\left(\frac{\Delta Re_3}{Re_3}\right)^2 = \left(\frac{\partial Re_3}{\partial \rho}\right)^2 \left(\frac{\rho}{Re_3}\right)^2 \left(\frac{\Delta\rho}{\rho}\right)^2 + \left(\frac{\partial Re_3}{\partial \rho}\right)^2 \left(\frac{Q}{Re_3}\right)^2 \times \left(\frac{\Delta Q}{Q}\right)^2 + \left(\frac{\partial Re_3}{\partial \rho}\right)^2 \left(\frac{L}{Re_3}\right)^2 \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\partial Re_3}{\partial \rho}\right)^2 \times \left(\frac{D}{Re_3}\right)^2 \left(\frac{\Delta D}{D}\right)^2 + \left(\frac{\partial Re_3}{\partial \rho}\right)^2 \left(\frac{\Delta P}{Re_3}\right)^2 \left(\frac{\Delta\Delta P}{\Delta P}\right)^2 \tag{9}$$

$$\frac{\Delta Re_3}{Re_3} = \sqrt{\left(\frac{\Delta\rho}{\rho}\right)^2 + 4\left(\frac{\Delta Q}{Q}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + 25\left(\frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta\Delta P}{\Delta P}\right)^2} \tag{10}$$

This equation emphasizes once more that the diameter error will contribute the most (five times) followed by the error in flow rate measurement (two times).

An error of 4.80% was obtained for the Reynolds number based on experimentally determined errors for all the individual parameters, using non-Newtonian fluids.

The loss coefficient, k_{con} (observed) is determined from Eq. (3) and as V_1 and V_2 are linked by β^2, the above equation may be

Table 1
All information obtained for contractions tested

D_u (mm)	D_d (mm)	β (D_d/D_u)	β^2 (A_d/A_u)	θ	C_{con}	k_{con}
42.3	9.28	0.219	0.048	−2.00	364 ± 108	1.00 ± 0.07
42.3	21.0	0.496	0.246	−1.88	288 ± 134	0.35 ± 0.08
42.3	36.0	0.851	0.724	−0.95	155 ± 109	0.17 ± 0.02

rewritten as

$$k_{\text{fitt}} = \left(\frac{2\Delta p_{\text{fitt}}}{\rho {V_2}^2}\right) + \theta \quad (11)$$

where

$$\theta = \alpha_1 \beta^4 - \alpha_2 \quad (12)$$

The highest expected loss coefficient error in terms of measured variables can then be expressed as

$$\frac{\Delta k_{\text{fitt}}}{k_{\text{fitt}}} = \pm\left(1 - \frac{\theta}{k_{\text{fitt}}}\right)\sqrt{\left(\frac{\Delta(\Delta p_{\text{fitt}})}{\Delta p_{\text{fitt}}}\right)^2 + \left(\frac{\Delta\rho}{\rho}\right)^2 + 4\left(\frac{\Delta Q}{Q}\right)^2 + 16\left(\frac{\Delta D}{D}\right)^2} \quad (13)$$

Eq. (13) suggests that the error in k_{fitt} increases as k_{fitt} becomes smaller, and conversely the error decreases as k_{fitt} is higher. The factor θ highlights both the influence of the velocity profile and the diameter ratio on k_{fitt}. It can be shown that θ is the highest when the pipes both run in laminar flow, and for Newtonian fluids, it takes the limiting values presented in Table 1.

The hydraulic gradients measured for water in the straight pipe sections were within 5% of the Colebrook-White [15] prediction, confirming the credibility of the instrumentation and the test rig.

3. Analysis of results

Pienaar and Slatter [22] showed that agreement amongst the work of researchers can be obtained if the same procedure is used when analysing the results. This section will only explain the final method used to analyse the pressure-drop results obtained from the pressure grade line approach. This approach involves the measurement of the pressure along the length of the pipes containing the contraction using several pressure transducers appose to measuring only the differential pressure difference across the contraction (Fig. 1).

The static pressure was measured along the length of the pipes, both upstream and downstream by means of eight pressure taps initially and after automation of the rig four and five either side of the contraction. The convention is to extrapolate the fully developed pressure gradient to the contraction plane, but in practice, it is often difficult to determine where the fully developed region is. For the evaluation of the results obtained in this work, points close to the contraction plane were excluded. This is shown in Fig. 2. The point closest to the contraction plane upstream was $24.4D_1$ to avoid distorted flow close to the contraction, and downstream was $57D_2$ to allow for fully developed flow after the contraction. This was also based on the trends observed from the pressure grade line versus axial distance plots for several fluids. This is especially important when dealing with low Reynolds numbers below 10, where significant deviations can be found [22]. The upstream conditions were sufficient for all flow regimes, but a downstream length of up to $100D_2$ is required for transitional flow [17]. The fact that $57D_2$ was

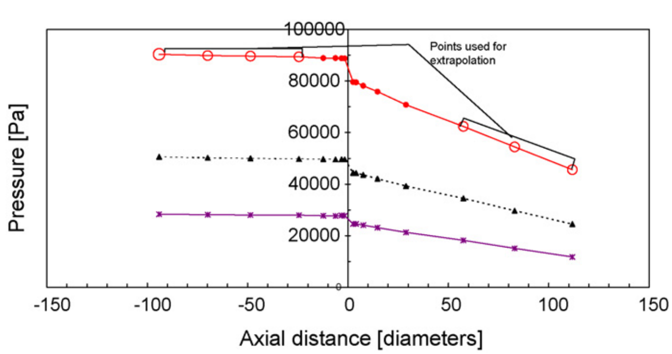

Fig. 2. Selection of data points for extrapolation.

used, could be the reason why such a scatter of results are obtained in the transitional area and more work should be done to investigate this in future, especially for non-Newtonian fluids.

The steps in calculating the loss coefficient were as follows:

- The slope and intercept were determined upstream and downstream using the selected data points as shown in Fig. 2. These data points were used for all tests.
- The pressure was calculated at the contraction plane both upstream and downstream.
- The pressure drop at the contraction (Δp_{con}) was determined by subtracting the downstream pressure from the upstream pressure at the contraction plane.
- The loss coefficient, k_{con}, was then calculated using Eq. (3).

For Newtonian fluids, α, the kinetic energy correction factor, was taken as 2 for laminar flow, for non-Newtonian fluids [23], where n is the flow behaviour index:

$$\alpha = \frac{3(3n+1)^2}{(2n+1)(5n+3)} \quad (14)$$

and for yield pseudoplastic fluids [24]:

$$\alpha = 2\tau_0^4 \frac{M(\tau_0 - \tau_y)^2 + B\tau_y(\tau_0 - \tau_y) + Y\tau_y^2}{[((\tau_0 - \tau_y)^2/(1+3n)) + (2\tau_y(\tau_0 - \tau_y)/(1+2n)) + (\tau_y^2/(1+n))]^3} \quad (15)$$

where

$$M = \frac{3}{(3n+1)(4n+2)(5n+3)}$$

$$B = \frac{6}{(2n+1)(3n+2)(4n+3)}$$

$$Y = \frac{1}{2(n+1)^3}$$

The laminar flow loss coefficient constant or Couette coefficient [25], C_{con}, is defined as the hyperbolic constant:

$$C_{\text{con}} = Re\, k_{\text{con}} \quad (16)$$

The loss coefficient constants, C_{con}, were evaluated using the logarithmic least square error:

$$E = \sum\left(\ln\left(\frac{C_{\text{con}}}{Re}\right) - \ln(k_{\text{con obs}})\right)^2 \quad (17)$$

C_{con} is obtained by minimising E for each contraction ratio.

4. Results and discussion

The loss coefficient data obtained for this work are presented below as plots of k_{con} versus Reynolds number. The Reynolds number in the downstream pipe was used in Figs. 6–8. For Newtonian fluids, the Newtonian Reynolds number [Re_{N}] has been used, for pseudoplastic fluids, the Metzner–Reed Reynolds number [Re_{MR}] and for yield pseudoplastic fluids, the Slatter Reynolds number [Re_3] [22].

Water, Newtonian oil, sugar solutions, CMC (3% and 5% by mass) and kaolin (5%, 8%, 10% and 13% by volume) were tested in the contractions. Rheological parameters were obtained from both the straight pipe tests and a rotational rheometer for comparison. The values of these parameters for kaolin tested are given in Table 2 as well as the shear rate range over which these were obtained. Although high shear rates could be obtained in the tube viscometer, rotational rheometers are prone to artefacts such as separation of the fluid and solid particles at higher shear rates. The data used for analysis of the rheological parameters were carefully selected to be free from such artefacts.

A comparison of the results shows excellent agreement within 5–10% between the rheometer and the tubes and is given in Fig. 3.

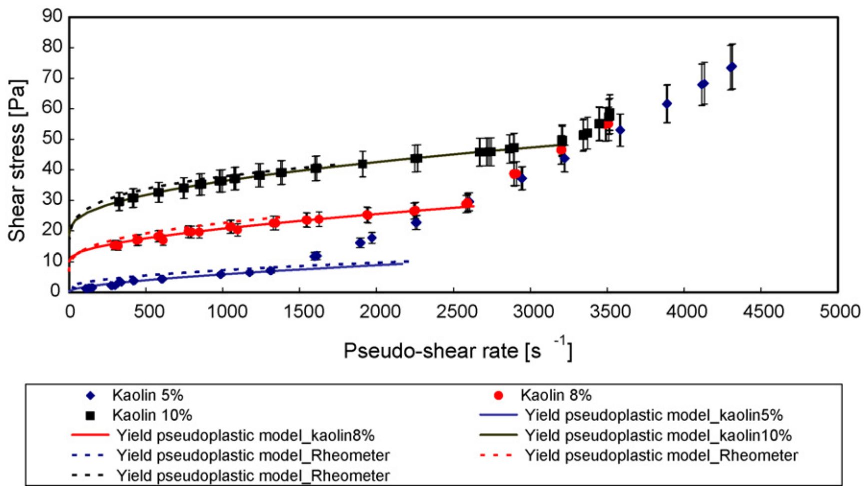

Fig. 3. Comparison of rheological parameters obtained in pipe loop and rheometer for kaolin tested in contraction $\beta = 0.22$.

Table 2
Rheological parameters obtained from rheometer for kaolin tested in contraction $\beta = 0.22$

Sample	τ_y (Pa)	K (Pa s^n)	n	$\dot{\gamma}$ region (s^{-1})
Kaolin 5%	1.23	0.147	0.514	10–632
Kaolin 8%	6.85	0.884	0.386	10–1262
Kaolin 10%	12.1	3.30	0.267	10–1262

Although turbulent flow is obtained in the tubes, only laminar flow data is used to obtain rheological parameters. The reason for this comparison was to ensure that even if there was insufficient laminar data in the pipes, the rheometer results could be used. As far as possible when sufficient laminar flow data could be obtained in the pipe, the rheological parameters obtained from the pipe tests were used for calculations.

The fluid viscous property range is summarised in Fig. 4 as a plot of apparent viscosity and shear rate which ranged from 0.01 to 20 000 Pa s over the shear rate range of 0.001–10 000 s^{-1}.

The effect of the rheology of the fluid and the Reynolds number can be been demonstrated for a kaolin sample using the Metzner–Reed Reynolds number and the Slatter Reynolds number (Re_3). The latter accounts for the presence of a yield stress. From Fig. 5 it is clear that dynamic similarity is attained only when using the Reynolds number that accounts for the yield stress, resulting in the same loss coefficient for kaolin and CMC. The use of Re_3 will give better prediction of fitting losses for fluids with a yield stress as shown in Fig. 4. However, even if the fluid does show the existence of a yield stress, but over the shear rate range of interest, the two-parameter power-law model adequately describe the rheological behaviour of the fluid over that range, Re_{MR} will be appropriate.

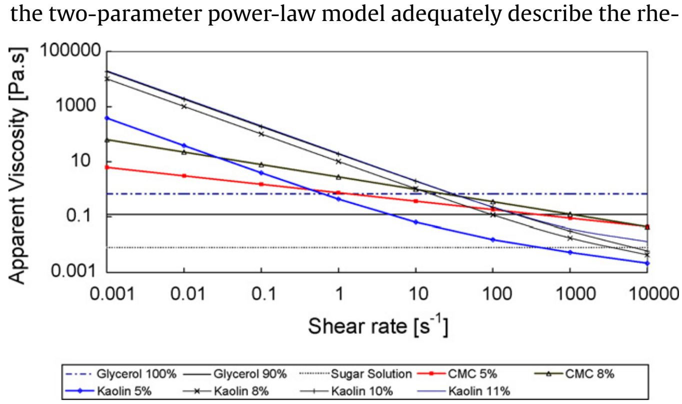

Fig. 4. Apparent viscosities for all fluids tested.

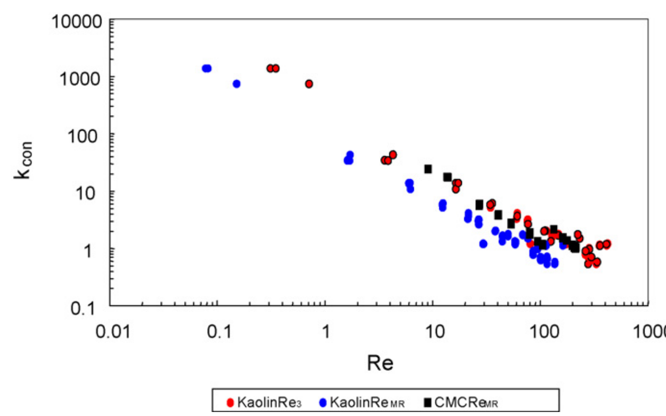

Fig. 5. Comparison of kaolin and CMC data using the appropriate Reynolds number to correlate loss coefficient.

The laminar loss coefficient constant, C_{con}, was obtained by means of Eq. (17), using k_{con} results at Reynolds numbers less than 10. Turbulent results were obtained at Reynolds numbers between 6000 and 70 000. The turbulent loss coefficient was obtained from the average turbulent values. The detailed results of the pressure drop along the axial distance across the contraction for all the fluids as well as the calculated variables are available in the database.

The experimental data were compared with semi-empirical models [5] and [17] and are given in Figs. 6–8. Some agreement was found with IHS ESDU [17] correlations in turbulent flow, but

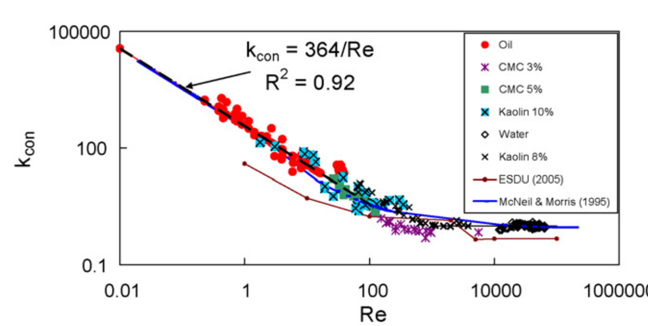

Fig. 6. $\beta = 0.22$ comparison of experimental data with IHS ESDU [17] and McNeil and Morris [5].

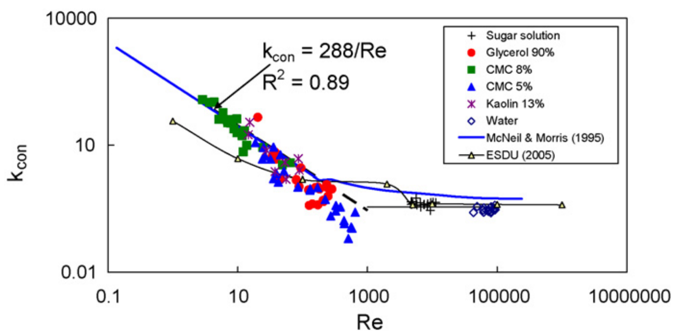

Fig. 7. $\beta = 0.5$ comparison of experimental data with IHS ESDU [17] and McNeil and Morris [5].

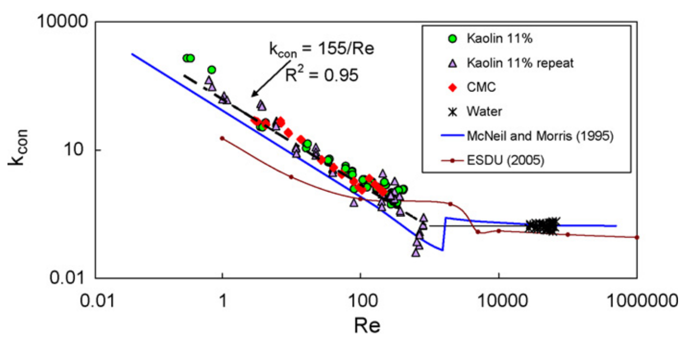

Fig. 8. $\beta = 0.85$ comparison of experimental data with IHS ESDU [17] and McNeil and Morris [5].

it under predicted laminar flow. Agreement was obtained with the model of McNeil and Morris [5], indicating the validity of the results. The deviation between experimental results and the McNeil and Morris model for $\beta = 0.85$ is due to the fact that this falls outside of the range of the geometry factor determined by them from the work of Edwards et al. [2] that was limited to $\beta = 0.66$. The new data will allow them to extend their model to this contraction ratio. McNeil and Morris [5] also predicts an earlier transition from the viscous to inertia driven range with decreasing contraction ratio and a very abrupt transition to turbulent flow for larger contraction ratios.

A summary of the results obtained are given in Table 1 and Fig. 9.The database of the results is available on the website of the Flow Process Research Centre of the Cape Peninsula University of Technology at:http://www.cput.ac.za/flowpro/contraction_data_may2006.zip.

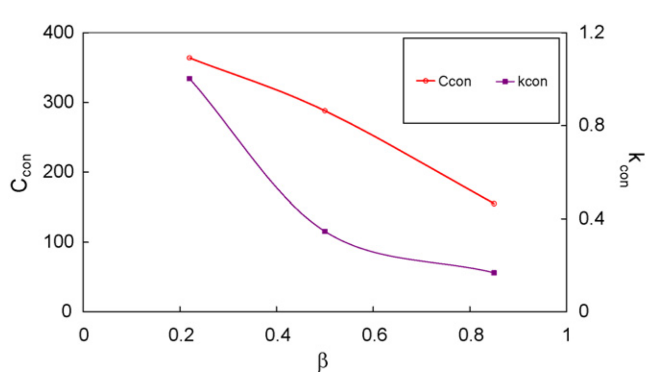

Fig. 9. Summary of loss coefficient data obtained in this work.

5. Conclusions

The pressure drop in three sudden contractions was measured using a range of Newtonian and non-Newtonian fluids. A detailed procedure for the analysis of the results is provided including the use of a kinetic energy correction factor for yield pseudoplastic fluids. The same loss coefficients can be used for both Newtonian and non-Newtonian fluids in laminar and turbulent flow, provided that the Reynolds number used properly accounts for the viscous properties of the fluid. IHS ESDU correlations agreed with some experimental data in turbulent flow, but under predicted laminar flow. Experimental work and mechanistic models agreed within the range of experimental data previously available. This is the first instance of a mechanistic analytical model agreeing with independent experimental data for a range of sudden contractions. This work has extended the database of results available over a wider range of Reynolds numbers and with more fluid types and contraction ratios. Agreement between our work, a mechanistic analytical model, and previous experimental work, speaks directly to the credibility of our results. This database is now available to those who are investigating losses through sudden contractions and need experimental results for comparison and validation.

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